

PRINCIPLES OF
ASSESSMENT MAPPING

LESSON II

ELEMENTS OF LAND SURVEYING

Ministry of Revenue
Assessment Standards Branch
1979/80

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The Assessment Standards Branch wishes to acknowledge the assistance and co-operation of the Land Surveys Office, Right of Way Branch, Ministry of Transportation and Communication, who supplied some of the material contained in this paper.

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ELEMENTS OF LAND SURVEYING

Part I: The Background

The Assessment Mapping staff will be duplicating, on their boards, the work of many Land Surveyors, over a number of years. It is useful to know something of the surveyors' techniques and problems, so as to understand the reasons for apparent discrepancies, and, on occasion, to locate survey monuments. However, it should be remembered that the profession of Ontario Land Surveyor involves years of training and an above average knowledge of mathematics; all that can be attempted here is a brief summary of a highly technical subject.

Surveying has been described as the art of making relatively large precise measurements with a maximum of accuracy and a minimum expenditure of time and labour. It also involves logical reasoning and personal judgment - the nature of the commodity being measured is such that no two projects are the same. Even a simple task of measuring the distance between two fences, involves judgment decisions; does one take the centre of the fence post? The face of the wire? A straight line that represents the average of a fence that is not straight? The purpose of the survey must also be considered. Surveying is costly, and the surveyor must balance time (ie. cost) against the value of the survey to his client.

Land surveying in Ontario dates from 1783 when steps were taken to provide for the arrival of United Empire Loyalists. An orderly system of land grants was necessary, and this meant that townships and lots had to be surveyed as quickly as possible.

However, the Government had very little money to spend on surveys. Land was cheap and plentiful, and there was no justification for careful and time consuming procedures. The cost of a precise survey would have been prohibitive; the intention was to provide the settlers with land quickly and at a minimum cost. A few feet or a few acres one way or the other were not important.

Lot corners were marked with wooden posts, which the surveyor made from any readily available materials. This practice prevailed from 1783 to 1903, the period in which 80 per cent of the township surveys were made.

These wooden 'monuments', of course, disappeared over the years, due to many causes, including fires and the clearing and settling of the land. In an effort to protect them, legislation was passed providing "that if any person or persons shall knowingly and wilfully pull down, deface, alter or remove any such monument so erected as aforesaid, he, she, or they, shall be adjudged guilty of felony and shall suffer death without benefit of clergy". (Even today, there is a maximum penalty of seven years imprisonment for removing a survey monument, yet they are disappearing constantly.)

Since 1903, iron bars have been planted at all survey points, major points being marked by bars four feet long by one inch square. In addition, such agencies as the Department of Highways use monuments of reinforced concrete with a bronze cap.

The problem, however, is that most of the original surveys have vanished. All that remains is a legacy of crooked fences that may or may not be on the line of the original, innaccurate survey! The modern surveyor has to

work within this doubtful framework, leaving a great deal to his personal judgment. Two surveyors may start from different points, and draw different conclusions from the existing features, producing different results where their work meets.

Most surveyors work from points established by earlier surveys, which, in turn, go back, generation after generation, to township surveys. As plans of subdivision gradually fill in the original lots, it is found that they do not fit together because the lots are not the size and shape that they are supposed to be! This is not to say that the original surveyors were in error. Each performed his work on the basis of his client's needs and the information available. Often, one of the first people to attempt to piece together all these independent plans and surveys, is the Assessment Draftsman.

The draftsman must try to visualize the surveyor, surrounded by trees, with broken down or non-existent fences, attempting to establish the original lot corners and lines of occupation, when none of the measurements match those of the original survey. He may find a road that should be sixty-six feet wide, yet has fences only fifty feet apart, or a lot corner that should be ninety degrees, and is actually eighty-eight degrees; a lot that contains only one hundred and ninety acres, may be occupied by two farmers, one of whom owns the north one hundred acres, and the other the south one hundred acres. In these, and many other cases, he has to make a personal decision.

When his decisions do not coincide with those reached by another surveyor half a mile or half a century away, the mapping draftsman should exercise his own discretion as to what information he portrays, accepts, or rejects.

Part II: Survey Measurements

The surveyor uses two basic reference directions - gravity and the meridian, or true north. Gravity is the basic reference for all measurements; strictly speaking, vertical means the direction of gravity and horizontal means perpendicular to gravity. Since the earth's surface is curved, the direction of gravity is different at every point on earth.

However, the difference is so small within a radius of about twelve miles, that it is disregarded in small surveys, and the surveyor, like the assessment draftsman, assumes the world to be flat. This type of survey is called a 'Plane Survey'. High precision surveys covering large areas take the spheroidal shape of the earth into account, utilizing spherical trigonometry, and these are called 'Geodetic Surveys'.

There are four types of measurement:

- (1) Horizontal Lengths
- (2) Horizontal Angles
- (3) Vertical Lengths
- (4) Vertical Angles

Only the first three will be discussed in this paper, since vertical angles are of little or no significance in assessment mapping.

(1) Horizontal Lengths:

Until late in the 19th Century, surveyors used the Gunter's Chain, which consisted of 100 iron links and measured 66 feet. The steel tape measure appeared in 1857, and although its general adoption was delayed by its relatively high cost, it eventually replaced the chain. However, the words chain, chaining, chainage, etc., have remained in the vocabulary of the surveying profession.

These tapes are normally divided into tenths and hundredths of a foot rather than inches (one hundredth of a foot is approximately $1/8$ "). The metric system is not yet in use in Canada, although it is appearing in England, Australia and New Zealand, and will very probably replace present systems of measurement.

The tape is held horizontally, high enough to clear obstacles, and plumb bobs are used to transfer the measurements to and from the ground. A tension handle, which resembles a spring balance, is often used to ensure that uniform tension is applied on all measurements. In addition, since the steel expands and contracts with changes in temperature, a thermometer may be attached to the tape, and a 'temperature correction' is made when the temperature varies from the standard of 68 degrees Fahrenheit.

In recent years, electronic measuring devices have appeared. The Geodimeter measures distances by recording the time required for a pulsating light beam to travel from a transmitter to a reflector and return. It can be used for distances of up to half a mile in daylight and further at night. For greater distances, there is the Tellurometer, which measures the time, in milli-microseconds, required for a radio microwave to travel from a transmitter to a remote unit and return. This device can measure distances up to forty miles.

However, devices such as these are of limited application in property surveys, if only owing to their high cost. Consequently, the steel tape will be around for the foreseeable future.

(2) Horizontal Angles:

All angles are measured with a theodolite or transit. Nowadays, the term 'theodolite' is generally reserved for high precision instruments, while the type commonly used is termed a 'transit'.

The Transit

The transit consists, basically, of a telescope, with cross hairs for sighting, mounted on both a horizontal and a vertical axis so that it can be turned in any direction in three dimensions. It has scales and verniers so that both vertical and horizontal angles can be established, and a compass is normally incorporated. It also has threads that screw onto a tripod, and there are levelling screws and spirit levels for bringing the instrument into a horizontal position. A plumb bob or a prismatic sight is used to position the transit accurately over a given point.

When the transit is set over a point and sighted on a second point, any angle may be measured from this line. If the direction of this first line is known, then the direction of any other line from the first point can be calculated quite easily, and thus any desired lines can be established.

This brings us to the second reference direction noted earlier - the meridian, or true north. While all directions are measured from north, there are two ways in which this is done - the azimuth and the quadrantal bearing.

Azimuths and Bearings

Azimuths are measured clockwise from north right around the circle to north again, as shown on the outer circle in Figure 1a. With quadrantal bearings, the circle is divided into four quadrants, N.E., S.E., S.W., N.W., each containing 90° . Bearings are read east and west from the meridian and can never exceed 90° . This is illustrated on the inner circle in Figure 1a.

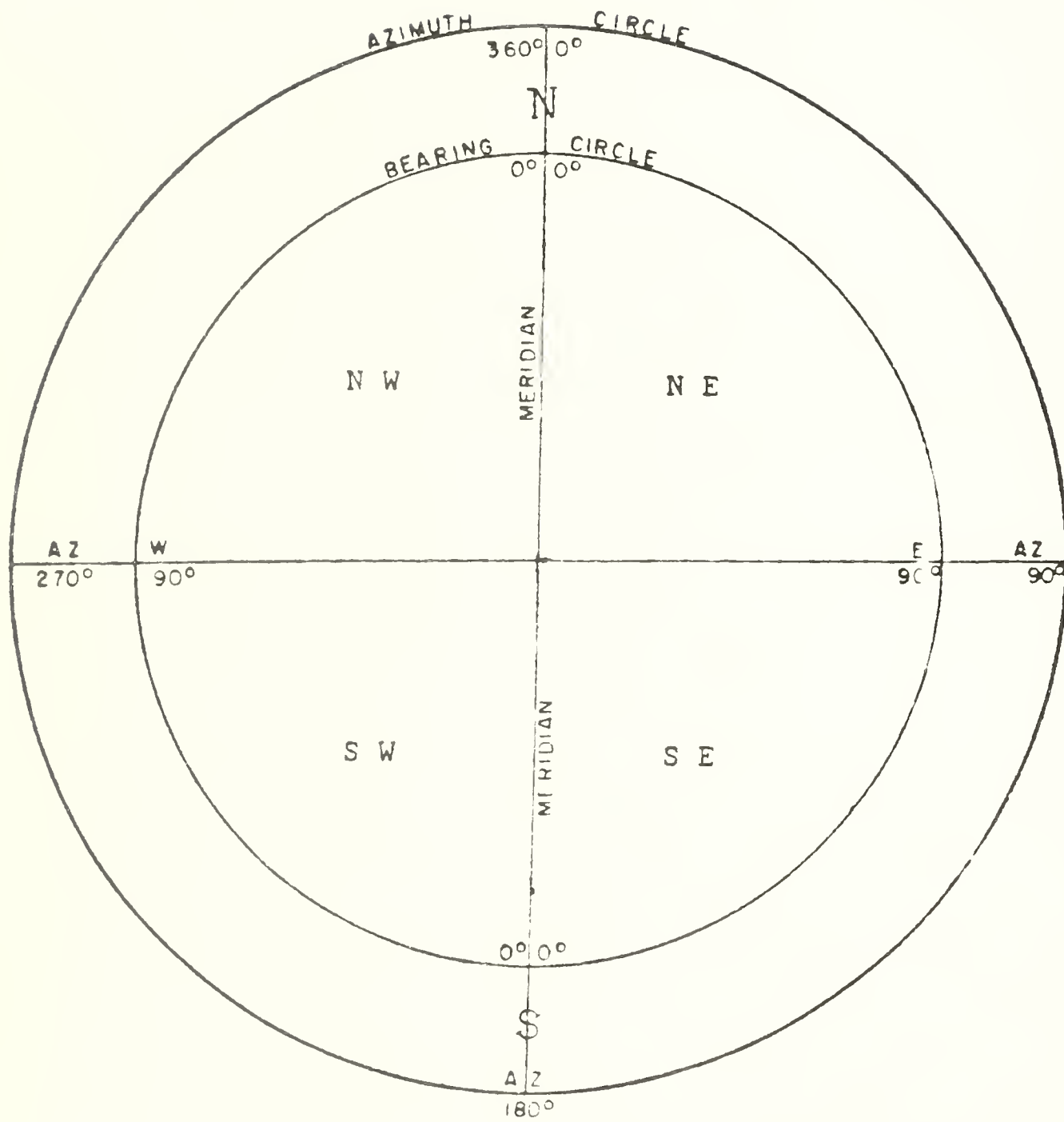


FIGURE 1a

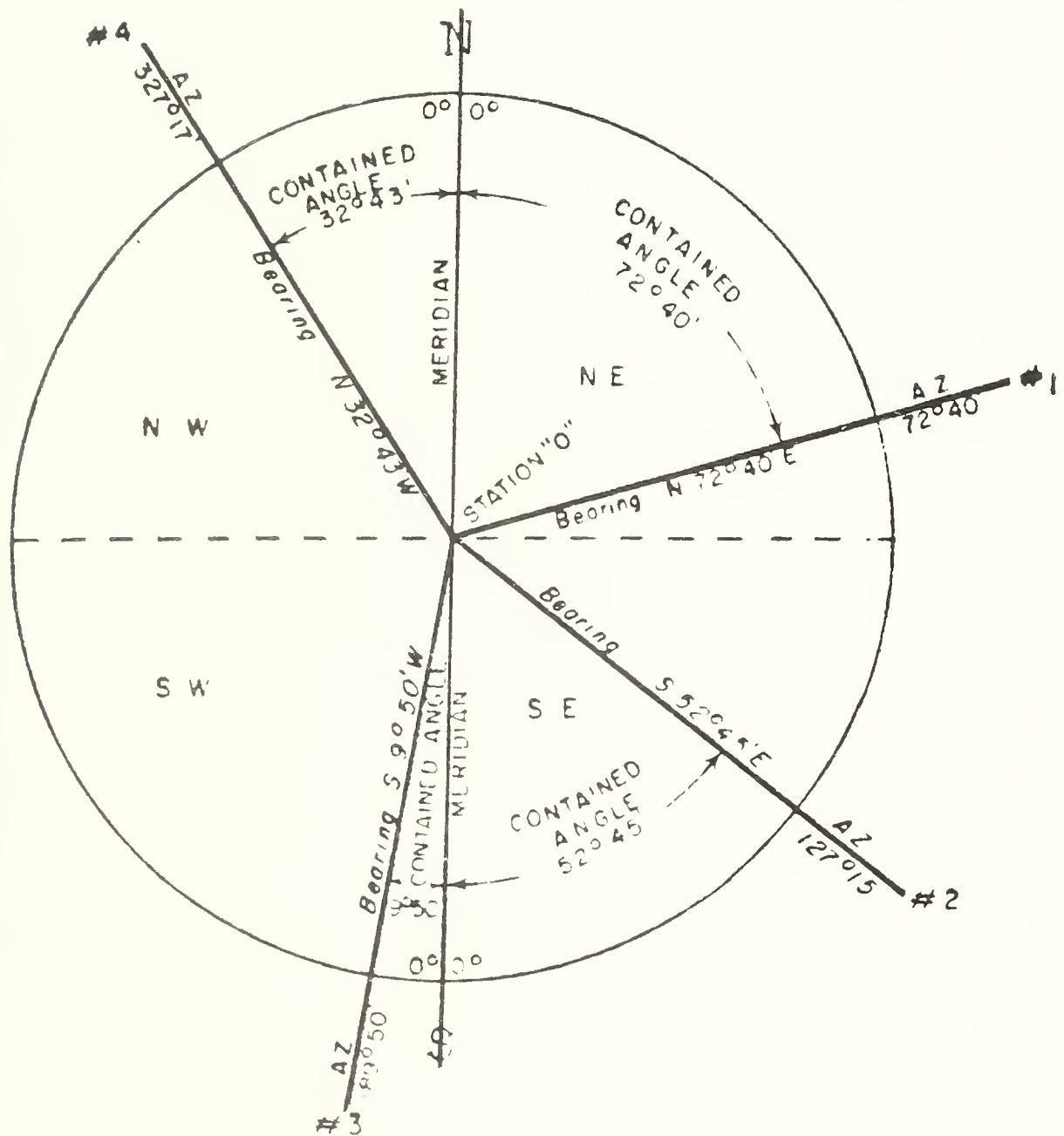


FIGURE 1b

Note that bearings within the NE quadrant have azimuths from 0° to 90° ; bearings within the SE quadrant have azimuths from 90° to 180° ; bearings within the SW quadrant have azimuths from 180° to 270° ; bearings within the NW quadrant have azimuths from 270° to 360° . Now to convert azimuths into bearings:-

By way of illustration let us assume that a surveyor is running a line due north, in other words, along the reference meridian, (see figure 1b). From station "O" he turns angles clockwise, to the right, $72^{\circ}40'$ to Point #1, (2) $127^{\circ}15'$ to Point #2, (3) $189^{\circ}50'$ to Point #3, and (4) $327^{\circ}17'$ to Point #4. These four angles, being read clockwise from true north, are actually the azimuths of lines from station "O" to Points #1, #2, #3, and #4. But it is the bearing of a line that is shown on a plan, not its azimuth. Hence the necessary conversion.

A glance at the figure makes it evident that any line having azimuth of less than 90° lies in the NE quadrant of the bearing circle and must have a NE bearing. Since the azimuth of a line in this quadrant is also the contained angle, the bearing of Line #1 (and all lines in the NE quadrant) is its azimuth with prefix N and suffix E, i.e., the bearing is $N72^{\circ}40'E$.

A line of azimuth between 90° and 180° just as evidently lies in the SE quadrant, as does Line #2. But, as the bearing of a line is the contained angle between it and the meridian, the bearing of Line #2 (and all SE quadrant lines) is $180^{\circ} - \text{azimuth} = 180^{\circ} - 127^{\circ}15' = S52^{\circ}45'E$.

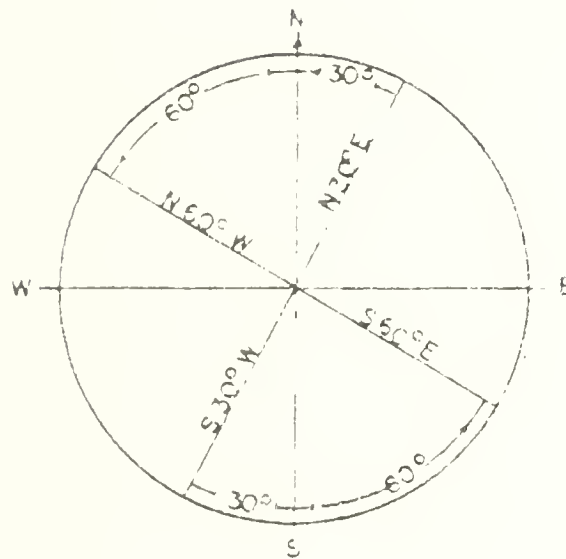
A line of azimuth between 180° and 270° lies in the SW quadrant. Again remembering the bearing to be the contained angle, the bearing of Line #3 (and all other SW quadrant lines) is $\text{azimuth} - 180^{\circ} = 189^{\circ}50' - 180^{\circ} = S9^{\circ}50'W$.

A line of azimuth between 270° and 360° lies in the NW quadrant of course. The bearing of Line #4 (and all NW quadrant lines) is $360^{\circ} - 327^{\circ}17' = N32^{\circ}43'W$.

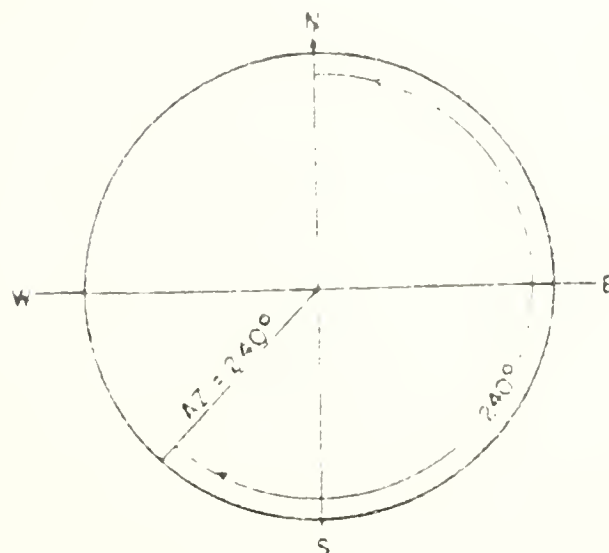
Remember then - NE quadrant bearing = azimuth; SE quadrant bearing = $180^\circ - \text{azimuth}$; SW quadrant bearing = $\text{azimuth} - 180^\circ$; and NW quadrant bearing = $360^\circ - \text{azimuth}$.

In Summary:

Quadrantal bearings are directions related to the north or south in an easterly or westerly direction through 90° (see diagram).



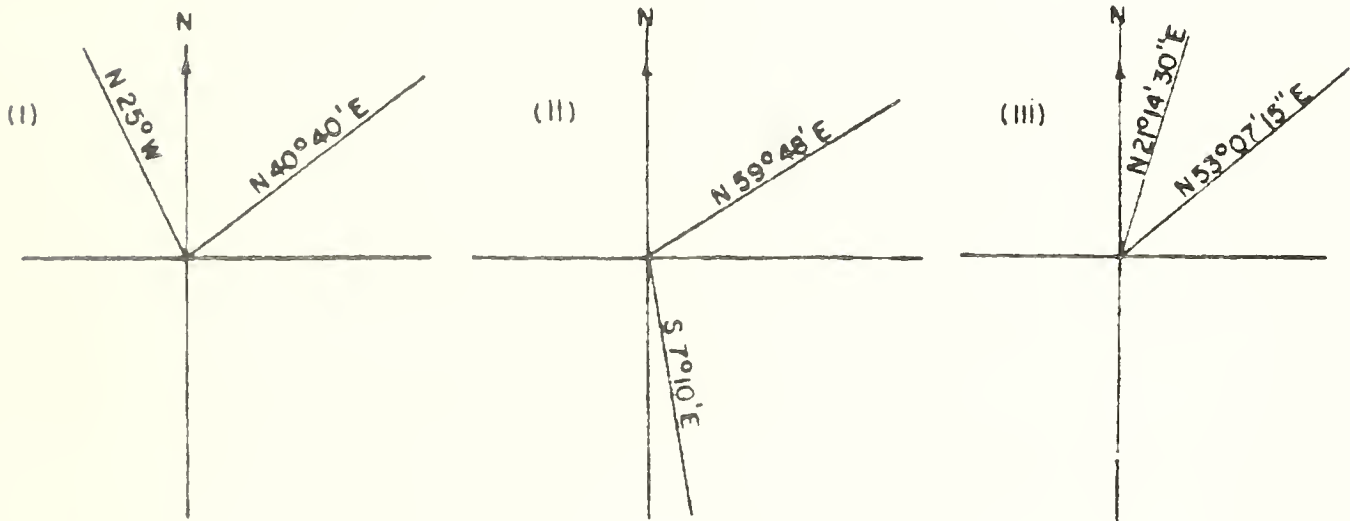
Azimuth bearings (azimuths), are directions related in a clockwise direction from north to the line (see diagram).



CONVERSIONS

Questions regarding bearings and azimuths may be of two types, or a combination thereof:

- (a) convert quadrantal bearings to azimuths, or vice versa
- (b) given lines and angles between them, compute the quadrantal bearings or azimuths from a given reference, or vice versa



(i) converting to azimuths $N40^{\circ}40'E$ is $40^{\circ}40'$

$N25^{\circ}W$ is $335^{\circ}00'$

the angle from N to $N40^{\circ}40'E$ is $40^{\circ}40'$

from N to $N25^{\circ}W$ is $25^{\circ}00'$

therefore the angle from $N25^{\circ}W$ to $N40^{\circ}40'E$ is $65^{\circ}40'$

(ii) converting to azimuths $N59^{\circ}48'E$ is $59^{\circ}48'$

$S7^{\circ}10'E$ is $172^{\circ}50'$

to find the angle between these lines, either

(a) having the azimuths, subtract the two; or

(b) produce one direction into the opposite quadrant, where

$S7^{\circ}10'E$ becomes $N7^{\circ}10'W$

or $N59^{\circ}48'E$ becomes $S59^{\circ}48'W$.

If we use $N7^{\circ}10'W$, then the angle between that and $N59^{\circ}48'E$ may be found by addition as in Example (1).

If we use $S59^{\circ}48'W$, then the angle between that and $S7^{\circ}10'E$ may also be found by addition

- (c) If we want the angle directly however, we first note that there is 180° between north and south. Then we note that $S7^{\circ}10'E$ represents the angle between that line and south, and that $N59^{\circ}48'E$ represents the angle between that line and north. If we add these together, and subtract the total from 180° , we are left with the required angle.

- (iii) Converting to azimuths $N53^{\circ}07'15"E$ is $53^{\circ}07'15"$
 $N21^{\circ}14'30"E$ is $21^{\circ}14'30"$
Subtracting the last from the first we get the angle between.

Simple problems: Find the azimuths, and the angles between:

1. $N82^{\circ}08'30"E$
 $S 1^{\circ}57'30"W$
2. $N18^{\circ}05'30"E$
 $S31^{\circ}31'30"E$
3. $N 4^{\circ}51'45"W$
 $S48^{\circ}39'15"W$
4. $S13^{\circ}12'22"E$
 $S42^{\circ}51'08"W$
5. $N81^{\circ}05'30"W$
 $N 6^{\circ}52'58"W$
6. $N13^{\circ}52'47"W$
 $S42^{\circ}42'42"W$
7. You are given a description of land in which the bearings are as follows:
the westerly side - $N14^{\circ}W$
the northerly side - $N58^{\circ}E$
the easterly side - $S24^{\circ}E$
the southerly side - $N76^{\circ}E$

- (a) make a sketch showing approximately the directions of each side
- (b) work out the four interior angles of the figure
- (c) convert each of the bearings to azimuths

8. You are given a description of land as follows:

N30°W 300 feet

East 450 feet

S30°W 300 feet

West 150 feet

- (a) find the interior angles of the parcel
- (b) convert the bearings into azimuths

(ANSWERS ON PAGE 34)

(3) Vertical Lengths:

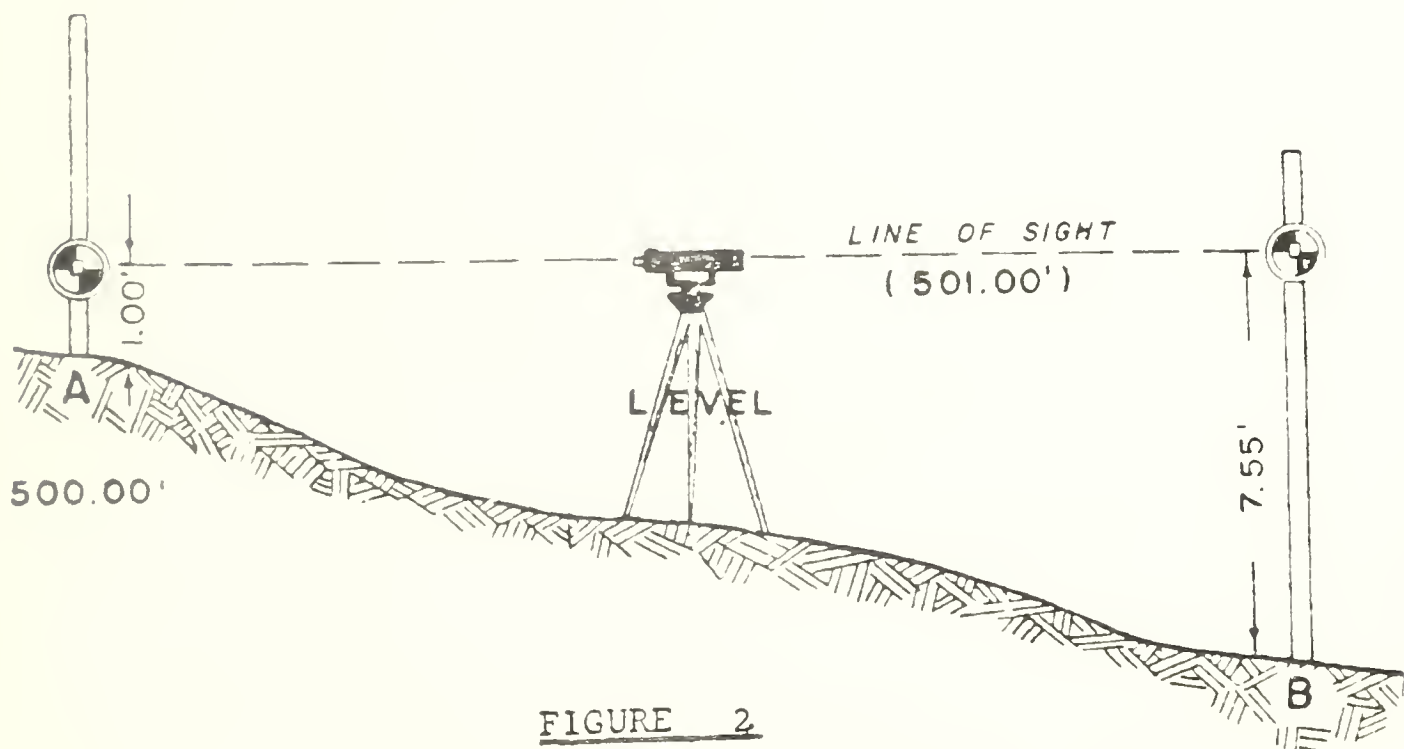
Vertical distances, or elevations, are measured with the aid of a level and a rod.

A level consists simply of a telescope with a cross-hair sight, a sensitive spirit level and adjusting screws so that the instrument can be brought into an exactly horizontal position. Like the transit, it is screwed onto

a tripod for use. There are also self-leveling instruments in which the line of sight is kept horizontal automatically.

Once the level is set properly, the line of sight of the telescope is horizontal - that is, at right angles to the direction of gravity. Measurements are taken with a wooden rod, graduated in hundredths of a foot, upward from zero at the bottom.

The theory of 'leveling' is quite simple. When the rod is placed on a point of known elevation, called a benchmark (B.M.), and read through the level, the height of the instrument above sea level is known. When the rod is placed on any subsequent point, and the reading subtracted from the height of the instrument, the elevation of that point can be established. (see Figure 2)



It is quite obvious that point 'B' is at an elevation of 493.45'.

The first reading, on the benchmark is called a backsight (B.S.). The B.M. plus the B.S. equals the height of the instrument (H.I.).

If the whole area under study cannot be covered from the one position, a point is established to serve the purpose of a secondary 'benchmark'. This is called a turning point (T.P.) The reading on this is termed a foresight (F.S.) and this is subtracted from the H.I. The instrument may then be moved and another B.S. taken on the T.P., and this could be continued as required.

Readings on points where the elevation is required, but which will not be used for subsequent backsights, are called intermediate sights (I.S.) and these too are subtracted from the H.I. The whole procedure is illustrated quite clearly in Figure 3.

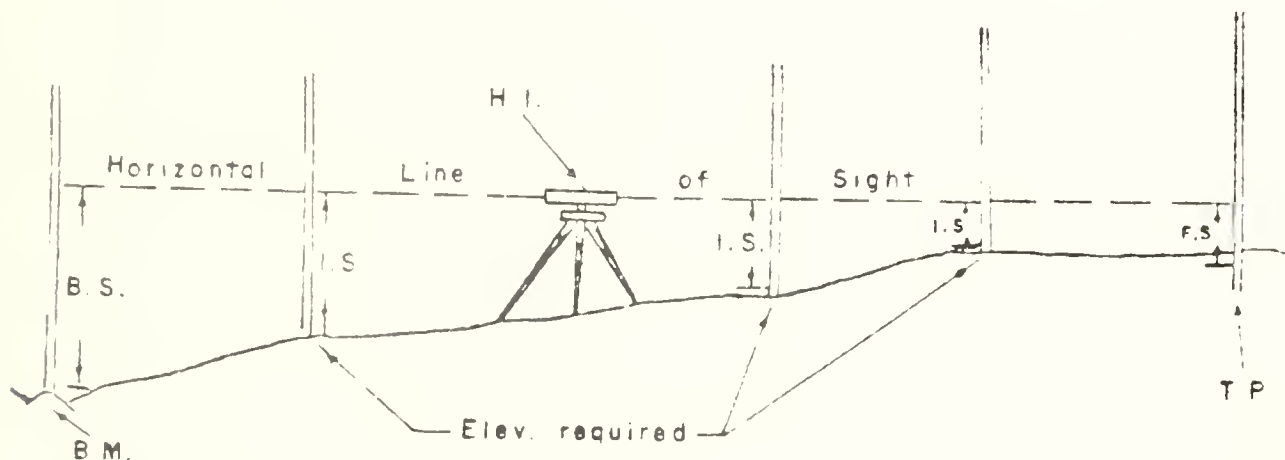


FIGURE 3

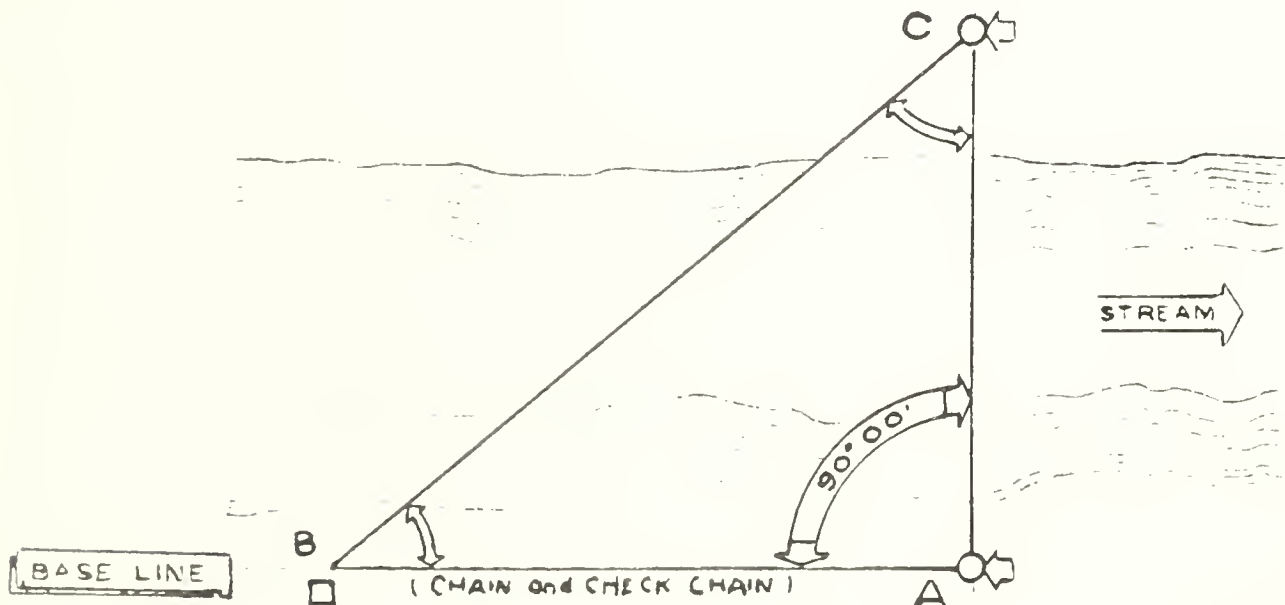
Part III: Basic Trigonometry

Obviously, a combination of horizontal distances and horizontal angles is all that is required to define property limits, so these two forms of measurement are of primary interest to the mapping draftsman. However, there is more to surveying and mapping than merely measuring angles and distances. It is also necessary, on occasion, to calculate missing distances or angles, so that frontages and areas may be computed accurately.

(1) Triangulation

The measurement of triangles, or trigonometry, is basic to surveying. The calculation of areas and missing distances can often be accomplished by the solution of triangles, and calculations involving more complex figures are developed from these basic techniques. The reader will undoubtedly be familiar with these formulae, but it is worthwhile reviewing them in this context.

The classic example in surveying is illustrated in Figure 4.



METHOD OF MAKING MEASUREMENTS BY TRIANGULATION

FIGURE 4.

In this case, the distance AC cannot be measured directly. The surveyor turns a 90° angle and measures a distance to a point B. He then measures the angle ABC, and, knowing all the angles and one side, a simple calculation will produce the distance AC.

Ratios

For any right angle triangle, the ratio of one side to another is a function of the angles. By means of a few simple formulae, missing distances or angles may be computed from known information.

The three terms, sine, cosine, tangent, are names given to equations involving the sides and angles of a right angle triangle. The side opposite the 90° angle is, of course, the hypotenuse, and the other two sides are called the adjacent and the opposite sides - depending on which angle is under consideration.

In Figure 5, looking at angle A, obviously side *a* is opposite and side *b* is adjacent. The opposite side divided by the adjacent side is called the tangent of an angle. Trigonometric functions are published in tables, so that if the angle A is known, its tangent may be readily determined. This makes it simple to calculate any one of the three figures by adjusting the formula. If angle A and side *a* are known, side *b* can be calculated by cross-multiplication.

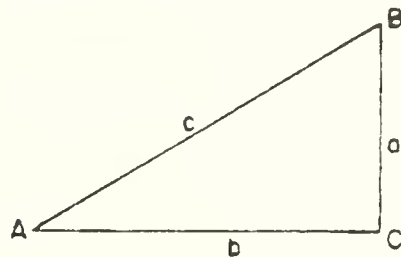
$$\text{If the tangent} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\text{then, tangent} \times \text{adjacent} = \text{opposite}$$

All that is required, is to look up the tangent of the angle A and multiply by the length of side b. The answer will be the length of side a, and this, of course, is the solution to Figure 4.

(see Book of Tables provided)

FIGURE 5



By ratios (dealing with angle A)

In the figure, the ratio:

$\frac{a}{c}$ is called the sine of A and written $\sin A$

$\frac{b}{c}$ is called the cosine of A and written $\cos A$

$\frac{a}{b}$ is called the tangent of A and written $\tan A$

$\frac{b}{a}$ is called the cotangent of A and written $\cot A$

$\frac{c}{b}$ is called the secant of A and written $\sec A$

$\frac{c}{a}$ is called the cosecant of A and written $\operatorname{cosec} A$

By reference to the side of a right triangle:

$$(1) \sin A = \frac{a}{c} = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$(4) \cot A = \frac{b}{a} = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$(2) \cos A = \frac{b}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$(5) \sec A = \frac{c}{b} = \frac{\text{hypotenuse}}{\text{adjacent side}}$$

$$(3) \tan A = \frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$(6) \operatorname{cosec} A = \frac{c}{a} = \frac{\text{hypotenuse}}{\text{opposite side}}$$

the first four are generally used.

Right Angle Triangles

Knowing the preceding formulae, a right triangle can be solved, when one angle and one side is known or when two sides are known.

Example: Given: Angle A, a (opposite side)
 Solve: b (adjacent side), c (hypotenuse)

From: $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$

$$\therefore \sin A \times \text{hypotenuse} = \text{opposite} \text{ (cross-multiplication)}$$

$$\therefore \text{hypotenuse} = \frac{\text{opposite}}{\sin A} \text{ (transposition)}$$

From: $\tan A = \frac{\text{opposite}}{\text{adjacent}}$

$$\therefore \tan A \times \text{adjacent} = \text{opposite}$$

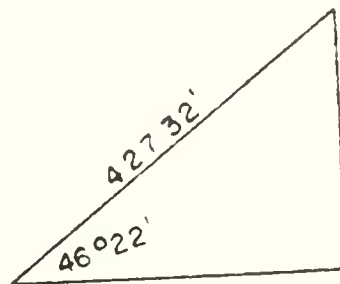
$$\therefore \text{adjacent} = \frac{\text{opposite}}{\tan A}$$

Example: Given: Angle A, hypotenuse
 Solve: adjacent side, opposite side
 From: $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$
 $\therefore \sin A \times \text{hypotenuse} = \text{opposite}$
 From: $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$
 $\therefore \cos A \times \text{hypotenuse} = \text{adjacent}$

Example: Given: Angle A, adjacent side
 Solve: opposite side, hypotenuse
 From: $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$
 $\therefore \cos A \times \text{hypotenuse} = \text{adjacent}$
 $\therefore \text{hypotenuse} = \frac{\text{adjacent}}{\cos A}$
 From: $\tan A = \frac{\text{opposite}}{\text{adjacent}}$
 $\therefore \tan A \times \text{adjacent} = \text{opposite}$

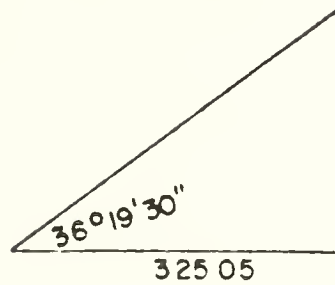
There are other equations but, we can conclude that, any series of equations can be used, as long as the given information relates to that particular equation.

EXAMPLE I



ANGLE	COS	SIN	H	ADJ	OPP	EQUATIONS
46°22'00"	.69004	.72377	427.32			
				294.87		ADJ = H x COS 46°22'00"
					309.28	OPP = H x SIN 46°22'00"

EXAMPLE 2



36°19'30"	.80566	.59236		325.05		
			403.45			H - ADJ ÷ COS 36°19'30"
				238.99		OPP = H x SIN 36°19'30"

Irregular Triangles

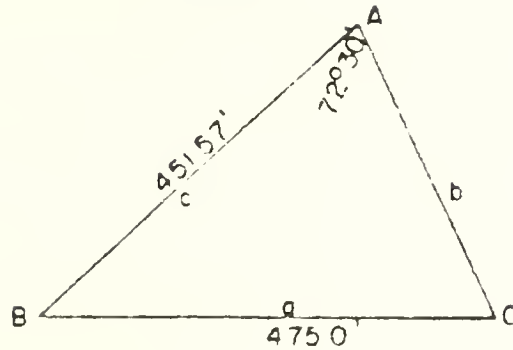
The preceding formulae apply only to right angle triangles. When dealing with irregular triangles - obtuse, scalene or equilateral - one solution is to drop a perpendicular to the base, creating two right angle triangles which can be easily solved.

However, a development of this is the Sine Law, written as follows:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

This formula may be used when two angles and one side are known, or two sides and the angle opposite one of them. When known information is inserted, the unknown may be readily determined by cross multiplication. This can be best illustrated by an example:

EXAMPLE



Two sides and an angle opposite to one of the sides

In A B C: Angle A and sides a and c are given
Solve: Angles B and C and side b

Solution: $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\therefore \frac{475.00}{\sin 72^{\circ}30'} = \frac{451.57}{\sin C}$$

$$\therefore 475.00 \times \sin C = 451.57 \times \sin 72^{\circ}30'$$

$$\therefore \sin C = \frac{451.57 \times \sin 72^{\circ}30'}{475.00}$$

$$\text{or } \sin C = \frac{451.57 \times .95371}{475.00}$$

$$\therefore \sin C = .90667$$

$$\therefore \text{Angle } C = 65^{\circ}03'$$

Because: Triangle Contains 180°
 $\therefore \text{Angle } B = 180^{\circ} - (\text{Angle } A + \text{Angle } C)$
 $\therefore \text{Angle } B = 180^{\circ} - 137^{\circ}33'$
 $\therefore \text{Angle } B = 42^{\circ}27'$

Solution: $\frac{a}{\sin A} = \frac{b}{\sin B}$

$$\therefore \frac{475.00}{\sin 72^{\circ}30'} = \frac{b}{\sin 42^{\circ}27'}$$

$$\therefore b \times \sin 72^{\circ}30' = 475.00 \times \sin 42^{\circ}27'$$

$$\therefore b = \frac{475.00 \times \sin 42^{\circ}27'}{\sin 72^{\circ}30'}$$

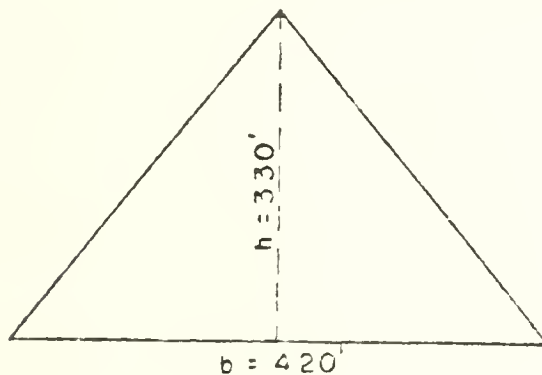
$$\text{or } b = \frac{475.00 \times .67494}{.95371}$$

$$\therefore b = 336.16$$

(2) Computation of Areas

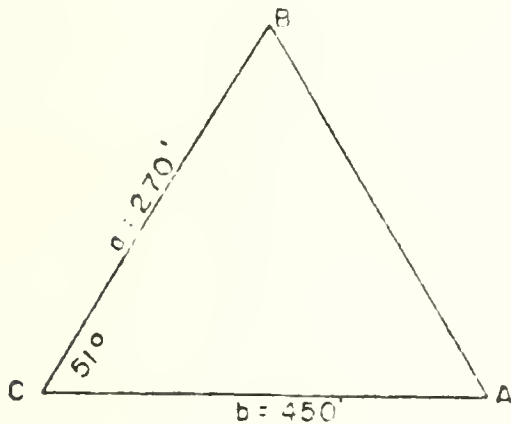
The assessment draftsman will often be called on to compute the area of irregular figures for which bearings and distances are known. From what we have discussed so far, a simple method, accurate enough for assessment purposes, would be to calculate the internal angles of the figure and then divide the figure into rectangles and triangles. From the formulae discussed earlier, we can easily compute the dimensions of these sub-figures and total their areas. The area of a triangle may be solved easily as follows:

AREA OF TRIANGLE



Given b and h

$$\begin{aligned} \text{Area} &= \frac{1}{2} b h \\ &= \frac{1}{2} (420) \times 330 \\ &= 210 \times 330 \\ &= 69300 \text{ sq. ft.} \end{aligned}$$

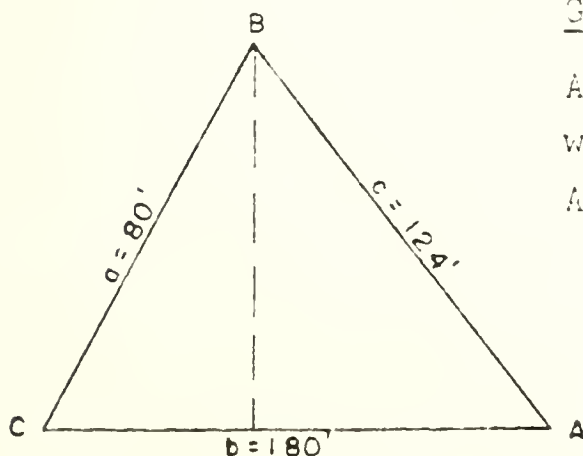


Given 2 sides and Contained Angle

$$\begin{aligned} \text{Area} &= \frac{1}{2} a b \times \sin C \\ &= \frac{1}{2} (270) \times 450 \times \sin 51^\circ \\ &= 135 \times 450 \times .77714 \\ &= 47211.620 \text{ sq. ft.} \end{aligned}$$

Check by solving for h, then using $\frac{1}{2} b h$

$$\begin{aligned} h &= 270 \times \sin 51^\circ = 209.83 \\ 225 \times 209.83 &= 47211.75 \text{ sq. ft.} \end{aligned}$$



Given 3 Sides No Angles

$$\begin{aligned} \text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ \text{where } s &= \frac{1}{2} \text{ sum of three sides} \\ \text{Area} &= \sqrt{192(192-80)(192-180)(192-124)} \\ &= \sqrt{192 \times 112 \times 12 \times 68} \\ &= \sqrt{17547264} \\ &= 4188.945 \text{ sq. ft.} \end{aligned}$$

This method of determining areas is common in assessment offices, but is primitive and time-consuming and would never be used by a surveyor. In surveying practice, areas are calculated by latitudes and departures.

Latitudes are distances north and south of a given point, and departures are distances east and west. North is plus, south is minus; east is plus, west is minus.

If a bearing is in the NE quadrant its latitude and departures will be plus and plus; in the NW quadrant, plus and minus, etc. By adding and subtracting latitudes and departures, a surveyor (or draftsman) can calculate areas very accurately and quite quickly.

(3) Surveyor's Method of Calculating Areas

The theory underlying this procedure is beyond the scope of this lesson. It is proposed to simply illustrate this technique by means of an example, which the assessment draftsman may use as a guide if desired.

We will assume that it is necessary to compute the exact area of a highway widening taken from the front of an irregularly shaped commercial property. A chart is set up showing the bearing of each line, followed by the cosine and sine of that angle, taken from the tables of trigonometric functions.

Next comes the distance, followed by the latitude and departure of each point, provision for what is called the double meridian distance, and finally a column for double areas. These last two columns will be discussed shortly.

LINE	BEARING	COSINE	SINE	DISTANCE	LATITUDE	DEPARTURE	D.M.D.'E	DOUBLE AREAS
A B	N 88° 53' E	01948	99981	330.91	+ 6.45	+ 330.85		
B C	S 8° 42' W	98849	15126	17.25	- 17.05	- 2.61		
C D	S 88° 53' W	01948	99981	277.36	- 5.41	- 277.31		
D A	N 72° 33' W	29987	95397	53.39	+ 16.01	- 50.93		
					+ 22.46	+ 330.85		
					- 22.46	- 330.85		
					00	00		

AREA = _____ SQ FEET

AREA = _____ ACRES

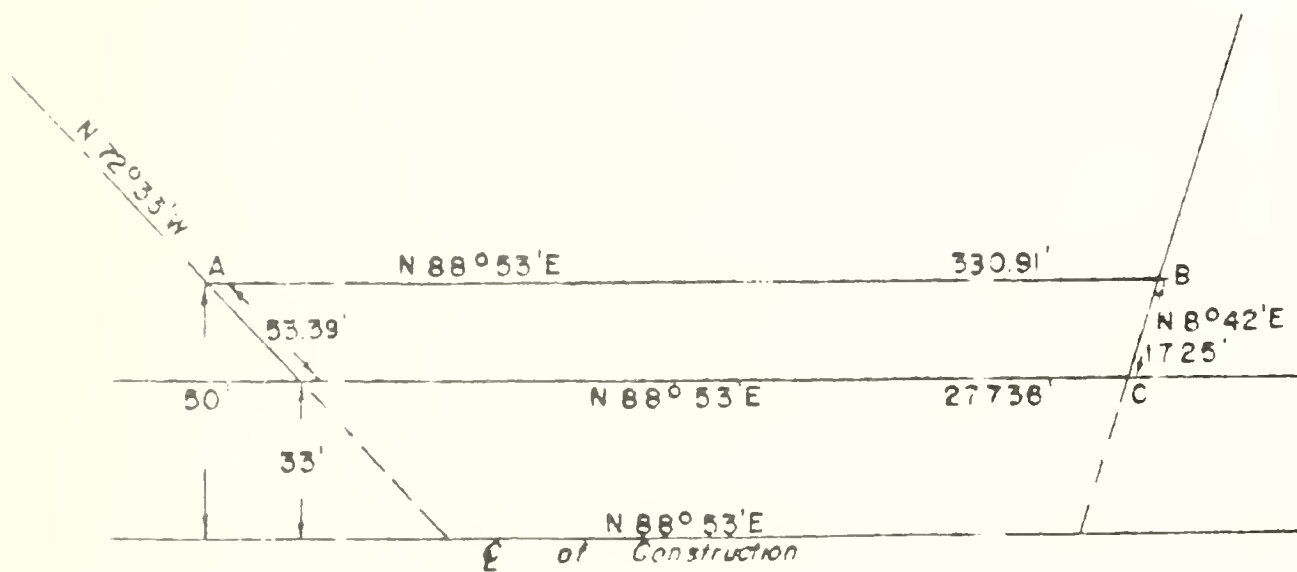


FIGURE 6

PROCEDURE

1. Label each corner of the parcel by a letter, commencing at the most westerly point and proceeding in a clockwise direction.
2. Record each line (AB, BC, etc.) in the first column.
3. Record the bearing of each line in the second column.
4. Obtain the cosine and sine of each angle from the tables and enter in the third and fourth columns respectively.
5. Enter the length of each line in the 'distance' column.
6. Label the latitude and departure columns plus or minus, depending on the bearing's quadrant: N.E. will obviously be plus and plus, S.E. will be minus and plus, S.W. will be minus and minus, and N.W. will be plus and minus.
7. Calculate the latitude of each line by multiplying the cosine by the distance.
8. Calculate the departure of each line by multiplying the sine by the distance.
9. As a check, total the plus and minus quantities for each column. They should be equal. If there is a major difference, you probably have the wrong sign on one figure.

Once the latitudes and departures have been determined, there are a number of ways of computing the area. The best known, and most popular, technique, is the Double Meridian Distance Method.

Double Meridian Distance Method of Area Computation

If a meridian was drawn through one point in the figure, a perpendicular from each of the other points to that meridian would produce a series of triangles and/or trapezoids. One could calculate the area of each of these, total them, and subtract those lying outside the boundaries of the figure. The area of each sub-figure would be determined by multiplying the height, which would be the latitude, by the average of the bases, which would be the perpendicular of the mid-point of each line.

This perpendicular from the mid-point of each line to the meridian is called the meridian distance. Multiplied by the latitude, it will produce the area of each sub-figure. Less arithmetic is required to find twice the meridian distance than the actual meridian distance, so the latitudes are multiplied by the double meridian distance (D.M.D.) and the area divided by two.

The D.M.D. of the first line is equal to the departure of that course.

The D.M.D. of each remaining course is equal to the D.M.D. of the preceding course plus the departure of that course plus the departure of the subject course.

The D.M.D. of the last course is equal to the departure of that course with the sign changed.

To return to our example:

LINE		LATITUDE	DEPARTURE	D.M.D.'S	DOUBLE AREAS
AB		+ 6.45	+ 330.85	330.85	+ 2134
BC		-17.05	- 2.61	659.09	-11237
CD		- 5.41	-277.31	379.17	- 205'
DA		+16.01	-50.93	50.93	+ 815
		+ 22.46	+ 330.85	AREA: -	13288: -
		- 22.46	- 330.85	AREA: +	2949: -
				DOUBLE AREA = 10339	
				AREA = $\frac{10339}{2}$ = 5170 SQ. FT.	
				= $\frac{5170}{43560}$ = 0.119 ACRES	

AREA = _____ SQ. FEET

AREA = _____ ACRES

PROCEDURE

10. Line AB: The D.M.D. is just the first departure = 330.85

11. Line BC: Double the first D.M.D. then add or subtract the second departure, depending on its sign

$$\begin{array}{r}
 = 661.70 \\
 - \quad 2.61 \\
 \hline
 659.09
 \end{array}$$

12. Line CD: The previous D.M.D. plus (or minus)
the departure of that course, plus
(or minus) the departure of the
subject course

$$\begin{array}{r}
 = 659.09 \\
 - \quad \underline{2.61} \\
 656.48 \\
 - \underline{277.31} \\
 \underline{\underline{379.17}}
 \end{array}$$

13. Line DA: Repeat

$$\begin{array}{r}
 = 379.17 \\
 - \underline{277.31} \\
 101.86 \\
 - \underline{50.93} \\
 \underline{\underline{50.93}}
 \end{array}$$

Which is the departure of the last course with
the sign changed.

14. Calculate the double areas by multiplying the
latitudes by the D.M.D.'s, obtaining the plus
or minus signs from the latitude column, and
dropping all fractions. (eg. 6.45×330.85
= 2134 square feet)

15. Total the plus double areas and the minus double
areas, and subtract the smaller from the larger.

16. Divide the difference by 2. The result is the area in square feet. Divide by 43,560 to reduce to acres. The draftsman may, as a check, apply the 'Rule of 23' often used by assessors: area in square feet x 23 = area in acres less 6 decimals.

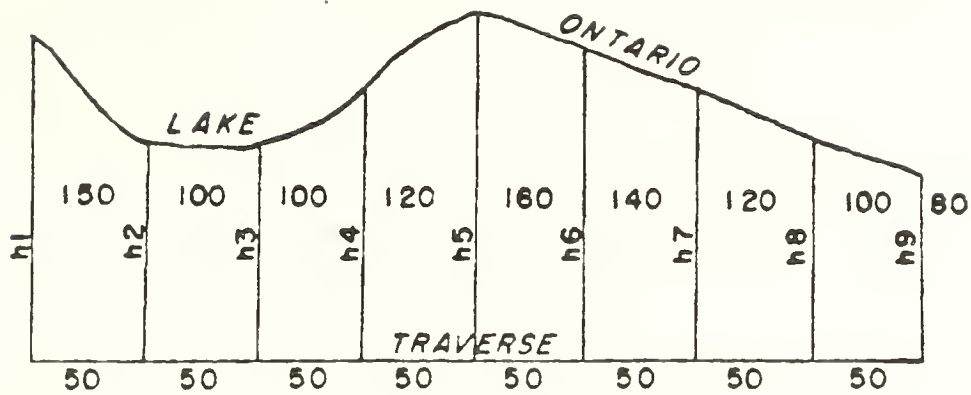
(4) Areas of Irregular Figures

Quite often, property boundaries follow irregular lines such as rivers and lakes. In this case, a surveyor will usually run a straight line roughly parallel to the boundary, and then take perpendicular measurements at set intervals to establish the location of the boundary. The area enclosed by the traverse can be calculated by one of two methods, 'Simpson's One-Third Rule' or the 'Trapezoidal Rule'.

Simpson's One-Third Rule

Find the sum of the end offsets, plus twice the sum of the odd intermediate offsets, plus four times the sum of the even intermediate offsets. Multiply the quantity thus determined by one third of the common interval between offsets, and the result is the required area.

EXAMPLE



$$\frac{50}{3} [h_1 + h_9 + 2(h_3 + h_5 + h_7) + 4(h_2 + h_4 + h_6 + h_8)]$$

$$\frac{50}{3} [150 + 80 + 2(100 + 160 + 120) + 4(100 + 120 + 140 + 100)]$$

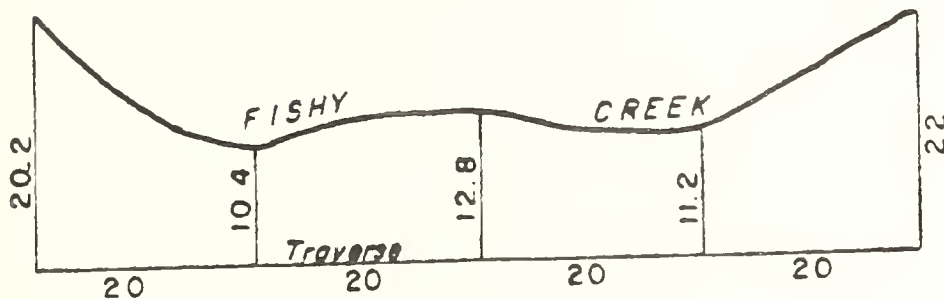
$$\frac{50}{3} \times (230 + 760 + 1840)$$

$$\frac{50}{3} \times 2830 = 47166 \text{ sq. ft.}$$

Use this rule only if there are an odd number of offsets.

TRAPEZOIDAL RULE

Add the average of the end offsets to the sum of the intermediate offsets. The quantity thus determined multiplied by the common interval between offsets is the required area.



$$20 \left[\left(\frac{20.2 + 22}{2} \right) + 10.4 + 12.8 + 11.2 \right]$$

$$= 20 \times 55.5$$

$$= 1110 \text{ sq. ft.}$$

Solving for area by above rule using the figure on previous page.

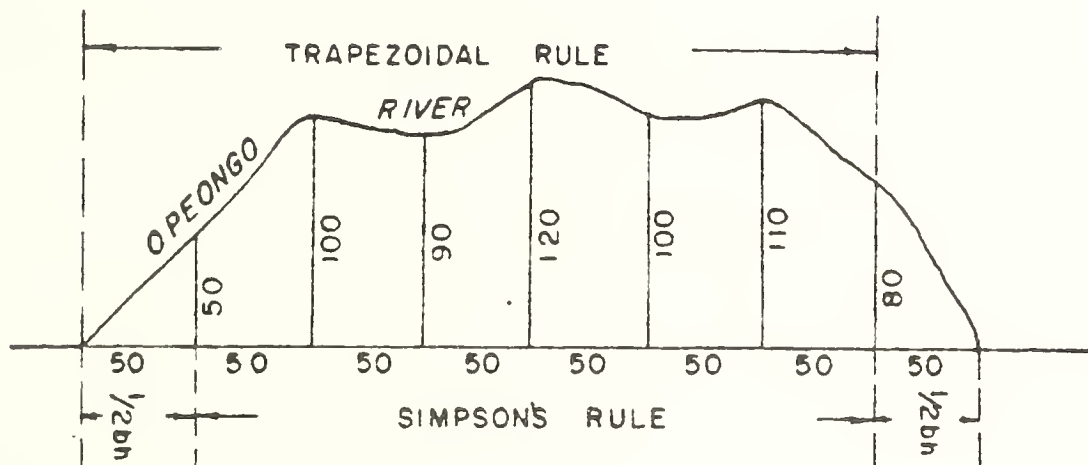
$$50 \left[\left(\frac{80 + 150}{2} \right) + 100 + 100 + 120 + 160 + 140 + 120 + 100 \right]$$

$$= 50 (115 + 840)$$

$$= 50 \times 955$$

$$= 47750 \text{ sq. ft.}$$

If and when the closed traverse of a irregular boundary appears like the figure below, i.e., traverse, and not the offsets, is tied to the lake, a combination of formula with either of the rules may be used.



Computation in 3 steps

Left Triangle: $\frac{1}{2} b h = 25 \times 50 = 1250 \text{ sq. ft.}$

Centre Section: $\frac{50}{3} [(50+80) + 2(90+100) + 4(100+120+110)]$

$$\frac{50}{3} (230+380+1320)$$

$$\frac{50}{3} \times 1830 = 30500 \text{ sq. ft.}$$

Right Triangle: $\frac{1}{2} b h = 25 \times 80 = 2000 \text{ sq. ft.}$

$\therefore$ Area of tract = $1250 + 30500 + 2000 = 33750 \text{ sq. ft.}$

Above problem may also be computed in two steps

$$50 (40+50+100+90+120+100+110)$$

$$50 \times 610$$

$$= 30500 \text{ sq. ft.}$$

$\therefore$ Area of Tract = $30500 + 2000 = 32500 \text{ sq. ft.}$

Planimeter

A useful device for measuring areas directly is the planimeter. A description of a typical planimeter is included in Appendix "A"; since any such device available in the drafting room will have instructions included, it need not be discussed further.

Needless to say, however, this instrument can only be as accurate as the drawing on which it is used. If the dimensions of the parcel are not accurately drawn, a planimeter is of limited value. If the dimensions are known precisely, the area could be computed by the method described previously. However, the planimeter has the advantage of speed.

It should be remembered that prints are prone to shrinkage or distortion, so should be checked prior to using this instrument. Normally, the area is read several times and the average taken, eg. the figure may be measured continually four times, the area read off at 39.32 acres $\div$ 4 = 9.83 acres.

Part IV: Topographic Surveys

It is often necessary to determine the dimensions and location of such features as buildings, rivers, lakes, ravines, etc. If the property is small, the simplest procedure is to measure perpendicular distances from measured points along the property lines.

On larger parcels, or where survey points are not readily accessible, it is customary to run a traverse. This consists simply of a series of lines, commencing at a known line, and proceeding via carefully measured bearings and distances so as to come close to all points of interest.

Measurements are taken, perpendicular to the traverse, to corners of buildings, river banks, etc. The draftsman can then plot the traverse, measure the appropriate distances along, and perpendicular to, the traverse, and plot physical features with a high degree of accuracy.

Surveyors frequently use a more sophisticated method known as Stadia. This employs a transit with two additional horizontal cross-hairs, called stadia hairs. Angles are turned from a base line to each point of interest. The distance is measured with the leveling rod by reading the difference between the upper and lower stadia hairs. Obviously, the farther away the rod is held, the smaller its image, so the greater the difference between the hairs.

Knowing the focal length of the telescope, and assisted by tables called Stadia Reduction Tables, the surveyor can calculate the distance from the transit to the rod. He can simultaneously note the elevation by using the regular cross-hair, and so can determine the information he requires with a minimum of field time. The telescope does not have to be level; he can read the vertical angles, and since he can calculate the distance to the rod, he can correct for the angle when calculating the elevations.

Finally, a very simple way of locating physical features is to simply set up a transit at first one end of a known line, such as a property line, and then at the other. The angle is read from the line to each point in turn, from both ends.

A draftsman with a protractor can draw the intersecting lines in minutes and plot the desired features with reasonable accuracy. Of course, for ravines or creeks, he would need an assistant to walk along the edge with a pole. This is probably the method most suited to studies made for assessment purposes.

A N S W E R S

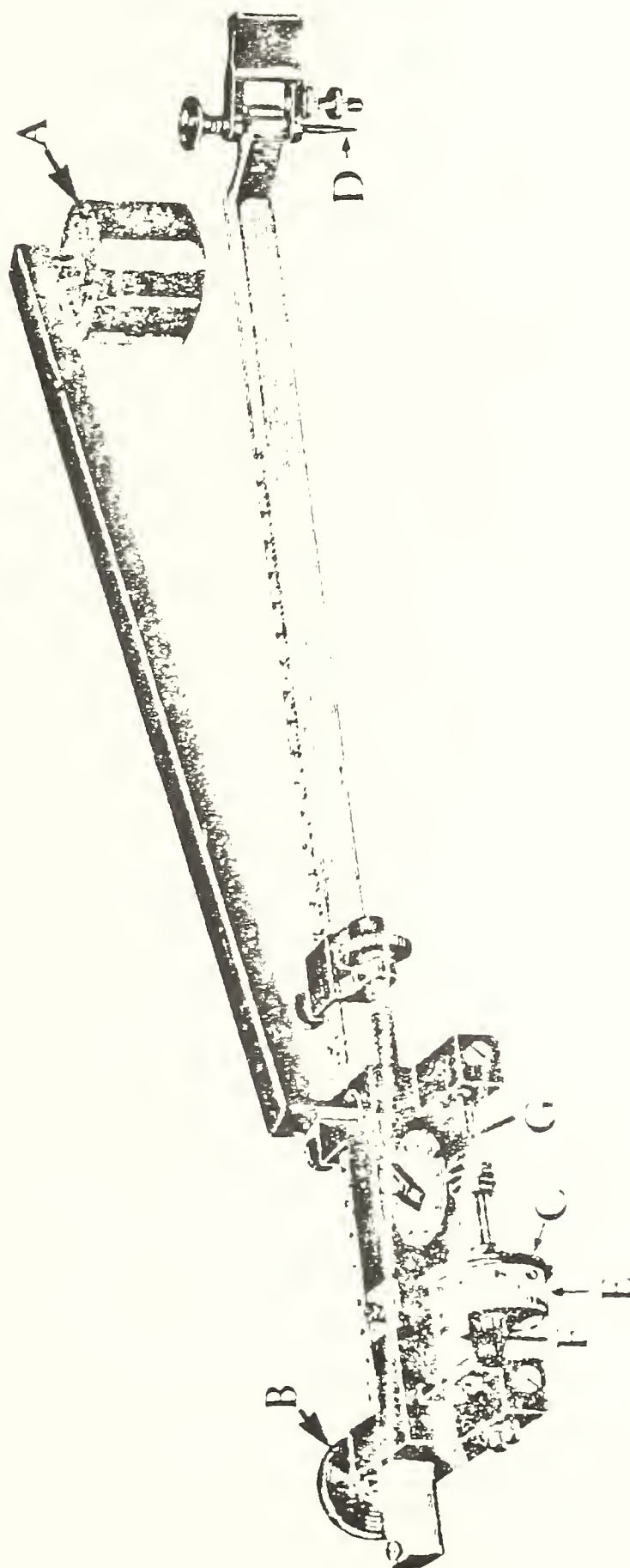
Page 11

- | | |
|---|---|
| 1. Az. - $82^{\circ}08'30''$
- $181^{\circ}57'30''$
Angle - $99^{\circ}49'$ | 4. Az. - $166^{\circ}47'38''$
- $222^{\circ}51'08''$
Angle - $56^{\circ}03'30''$ |
| 2. Az. - $18^{\circ}05'30''$
- $148^{\circ}28'30''$
Angle - $130^{\circ}23'$ | 5. Az. - $278^{\circ}54'30''$
- $353^{\circ}07'02''$
Angle - $74^{\circ}12'32''$ |
| 3. Az. - $355^{\circ}08'15''$
- $228^{\circ}59'15''$
Angle - $126^{\circ}29'$ | 6. Az. - $346^{\circ}07'13''$
- $222^{\circ}42'42''$
Angle - $123^{\circ}24'31''$ |

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7. (b) 108° , 82° , 80° , 90°
(c) West boundary, Az. - 346°
North boundary, Az. - 58°
East boundary, Az. - 156°
South boundary, Az. - 76°
8. (a) 60° , 60° , 120° , 120°
(b) West boundary, Az. - 330°
North boundary Az. - 90°
East boundary, Az. - 210°
South boundary Az. - 270°

ILLUSTRATION



COMPENSATING PLANIMETER

COMPENSATING PLANIMETER

Planimeters are often graduated to read directly in acres at a scale of $1" = 100'$.

The Planimeter is supported at the

- (a) anchor point "A",
- (b) Wheel "B",
- (c) Roller "C", and
- (d) Tracing Pin "D".

The circumference of the drum "E", attached to the roller "C", is graduated into 100 equal parts.

The Vernier, "F", is graduated into 10 equal parts.

By means of a worm-gear the roller turns the graduated disc "G" at a ratio of 10:1.

Each graduation on disc "G", indicates one complete revolution of drum "E", and represents 10 acres.

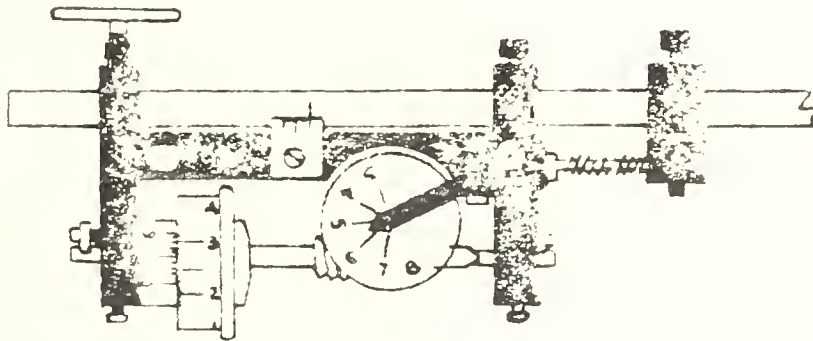
Acres and tenths of acres are read on drum "E" and hundredths of acres are estimated on Vernier "F".

OPERATING PROCEDURE

- (1) The plan from which the area is taken must be flat.
- (2) Press the needle tip of anchor point "A" into the paper at a point outside the figure, so that the entire figure can be traversed by the tracing pin "D".
- (3) Set the tracing pin at a starting point on a perimeter of the figure.
- (4) Set the drum to read zero or record the reading on the drum.
- (5) Traverse the perimeter of the figure in a clockwise direction terminating at the starting point.

- (6) Take the final reading.
- (7) The difference between the initial and the final readings is the positive area of the figure in acres and decimals of acres provided the scale of the plan is $100' = 1''$.

Example



The reading is 52.27 acres.

If areas are taken from a plan at a scale other than $100' = 1''$, the following table is a guide for converting planimeter readings to the proper acreage.

Table

<u>Scale of plan</u>	<u>Proper acreage</u>
$1'' = 200'$	Multiply by 4
$1'' = 300'$	Multiply by 9
$1'' = 400'$	Multiply by 16
$1'' = 50'$	Divide by 4
$1'' = 40'$	Divide by $6 \frac{1}{4}$

The table is determined by a direct ratio of the scales. Because the measurement of acreage is a square measurement and the planimeter

is calibrated to read at a scale of $100' = 1''$, therefore, the ratio is as follows:

$$1 \text{ is to } x \text{ as } (100)^2 \text{ is to } (\text{scale})^2$$

$$\text{e.g. scale of plan is } 50' = 1''$$

Determine the constant "X" to be applied to the reading.

Solution:

$$\frac{1}{x} = \frac{100^2}{50^2}$$

$$\frac{1}{x} = \frac{10000}{2500}$$

$$x(10000) = 1(2500)$$

$$x = \frac{2500}{10000}$$

$$x = 1/4$$

ergo: divide the reading by 4

APPENDIX B

UNITS OF LENGTH AND OF SURFACE

LENGTH

United States and British Standards

1 inch = 0.083333 foot = 0.027778 yard

1 foot = 12 inches = 0.333333 yard

1 yard = 36 inches = 3 feet

1 rod = 16.5 feet

1 Gunter's chain = 66 feet = 4 rods = 0.012500 mile

1 engineer's chain = 100 feet = 0.018939 mile

1 mile = 5280 feet = 1760 yards = 320 rods = 80 Gunter's
chains.

SQUARE MEASURE

United States and British Standards

144 sq. inches = 1 sq. foot

9 sq. feet = 1 sq. yard

10 square chains (Gunter's) = 1 acre

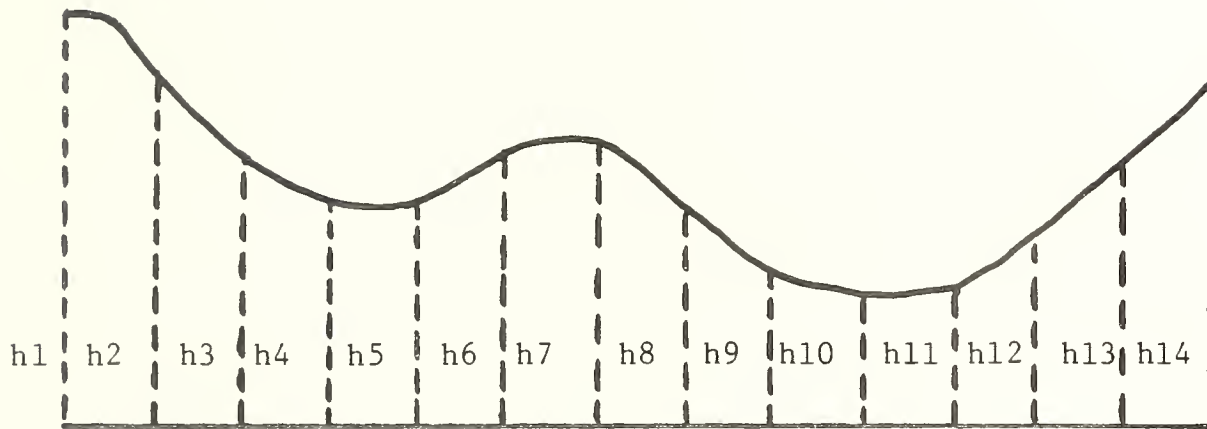
640 acres = 1 square mile

43560 sq. ft. = 1 acre

INCHES REDUCED TO DECIMALS OF A FOOT								
INCHES	0	1/8"	1/4"	3/8"	1/2"	5/8"	3/4"	7/8"
0	0000	0104	0208	0313	0417	0521	0625	0729
1	0833	0938	1042	1146	1250	1354	1458	1563
2	1667	1771	1875	1979	2083	2188	2292	2396
3	2500	2604	2708	2813	2917	3021	3125	3229
4	3333	3438	3542	3646	3750	3854	3958	4063
5	4167	4271	4375	4479	4583	4688	4792	4896
6	5000	5104	5208	5313	5417	5521	5625	5729
7	5833	5938	6042	6146	6250	6354	6458	6563
8	6667	6771	6875	6979	7083	7188	7292	7396
9	7500	7604	7708	7813	7917	8021	8125	8229
10	8333	8438	8542	8646	8750	8854	8958	9063
11	9167	9271	9375	9479	9583	9688	9792	9896

ASSIGNMENT LESSON II - EDITION TEN CONT'D

4.

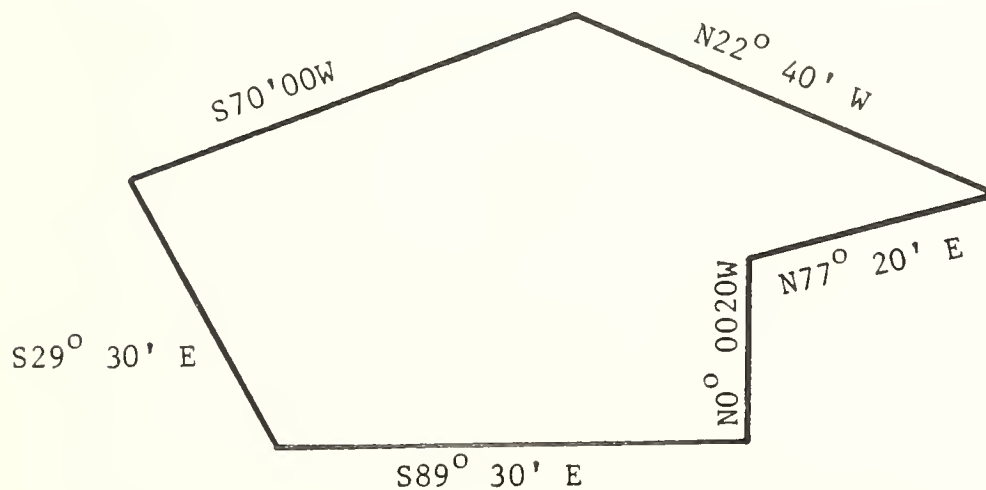


Interval dist. of 20 cm.

Calculate area in sq. cms.

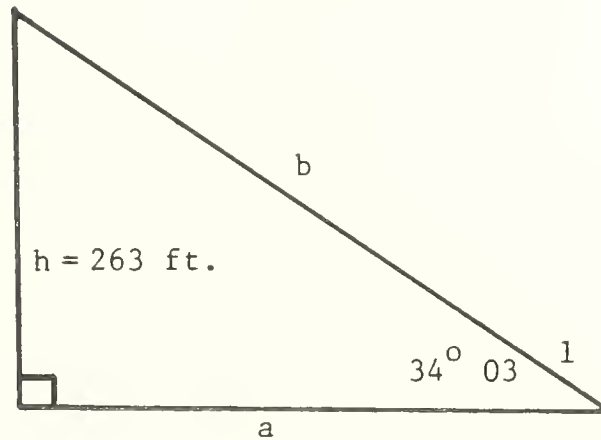
h1 = 55	h8 = 30
h2 = 47	h9 = 21
h3 = 37	h10 = 17
h4 = 30	h11 = 18
h5 = 30	h12 = 25
h6 = 38	h13 = 35
h7 = 38	h14 = 45

5. Determine the interior angles for the following figure:



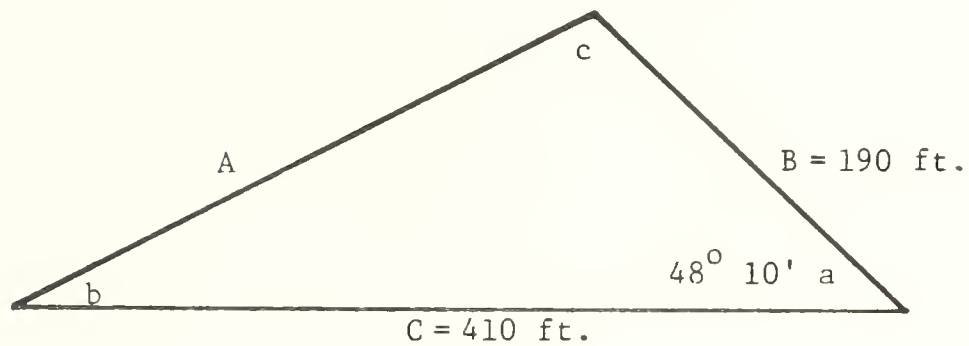
ASSIGNMENT LESSON II - EDITION TEN

1.



Calculate the area of the triangular lot.

2.



Calculate angle c and the distance of A.

3. Given Azimuth - Find the bearing of each line and angles between them:

(a) $23^{\circ} 43' 54''$
 $46^{\circ} 34' 55''$

(d) $47^{\circ} 03' 47''$
 $328^{\circ} 47' 07''$

(b) $33^{\circ} 50' 00''$
 $174^{\circ} 01' 57''$

(e) $257^{\circ} 34' 09''$
 $346^{\circ} 58' 43''$

(c) $11^{\circ} 11' 11''$
 $111^{\circ} 51' 13''$

Ontario. Ministry of Reven
Elements of land surveying
fnkk
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